PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Tidal Studies from the Perturbations in Satellite Orbits

Anny Cazenave, Sylviane Daillet and Kurt Lambeck

Phil. Trans. R. Soc. Lond. A 1977 **284**, 595-606
doi: 10.1098/rsta.1977.0036

Phil. Trans. R. Soc. Lond. A. **284**, 595–606 (1977) [595]
Printed in Great Britain

Tidal studies from the perturbations in satellite orbits

By Anny Cazenave,† Sylviane Daillet† and Kurt Lambeck‡

† *Groupe de recherche de Géodésie Spatiale, Centre National d'Etudes Spatiales*

‡ *Institut de Physique du Globe, Université Paris VI, and Département des Sciences de la Terre, Université Paris VII.*

Tidal perturbations in the orbits of close Earth satellites permit the Love number k_2 and the phase lag ϵ_2 of the tidal effective Earth to be estimated. These parameters differ significantly from the nominal values of $k_2 = 0.30$ and $\epsilon_2 < 0.5°$ that would be expected if only the solid tide was important. This difference is due to the contribution of the oceans to the total tidal potential. This contribution is less well known than the solid tide potential and the study of the orbital perturbations permits an estimation of some of the long wavelength variations in the ocean tide. In this paper we discuss the method and the results obtained from two satellites for the S_2 and M_2 ocean tide. These tidal parameters are compared with and combined with numerical models of these tides to give improved parameters to be used in any orbital theory.

Introduction

The periodic tidal deformations of the Earth give rise to small but significant perturbations in the motions of close satellites as pointed out already by Kaula (1962). First attempts at analysing orbits for these perturbations were reported by Newton (1965, 1968) and Kozai (1968). The former, in 1968, analysed Doppler observations of four satellites collected by the Tranet network and the latter analysed camera observations of three satellites collected with the Smithsonian Baker–Nunn network. Although there was considerable dispersion between their estimates of the Love number k_2 and phase lag ϵ_2 from the orbits of different satellites their results appeared to be in general agreement with those obtained by more conventional methods such as gravimetric and tilt tide observations or from the rotation of the Earth. More recent studies by Anderle (1971), Smith, Kolenkiewicz & Dunn (1973) and Douglas, Klosko, Marsh & Williamson (1974), however, resulted in values for the Love numbers k_2 of 0.25 and even smaller. Lambeck & Cazenave (1973) pointed out that this apparently aberrant result was mainly the consequence of the neglect of the ocean tide in these earlier studies. They showed (see also Lambeck 1973) that the effect of neglecting the ocean tide would be to introduce errors in the Love number determinations of up to 15 % and in the phase lags of several degrees depending on the ocean tide component and on the orbit of the satellite. Composite Love numbers, including solid Earth and ocean contributions, will therefore depend on the orbital parameters of the satellite investigated as well as on the frequency of the tidal component. This latter dependence makes a generalized development of Love numbers in terms of latitudinal and longitudinal dependencies, as attempted by Kaula (1969), unwieldy and it appears preferable to develope an explicit orbital theory for the ocean tide potential. Lambeck, Cazenave & Balmino (1974) developed such a theory and gave a general review of the solid and fluid tidal effects on close Earth satellite orbits. Recent estimates of ocean tide parameters using this development have been given by Felsentreger, Marsh & Agreen (1976).

Precise analysis of satellite orbits for tidal parameters is of interest for several reasons. First, the perturbations can be quite significant, reaching several seconds of arc in some cases. They must therefore be known and it must be possible to model them in any orbital computation if we wish to study other, smaller, perturbations due to, for example, polar motion. Secondly, the ocean tide parameters that can be estimated from the orbital perturbations describe a global wavelength in the tidal expansion whereas terrestrial observations, be it directly from tide-gauge observations or indirectly by measuring the ocean loading of the continents with the aid of gravimeters or pendulums, describe local variations in the ocean tide. Thus the terrestrial and satellite methods are two complementary measurements to be considered in the modelling of the world's tides. The possible impact of the satellite results on the numerical tide models is discussed further below. Thirdly, the tidal parameters affecting the satellite motion also act on the lunar orbit and provide a direct estimate of the present secular evolution of the size, shape and inclination of the lunar orbit (Lambeck 1975).

Satellite methods do suffer from one serious limitation, namely that the M_2 tide, the most important on the Earth and therefore of great interest to physical oceanographers and to students of the evolution of the Earth–Moon system, often has an almost insignificant effect on the satellite orbits, whereas other tidal frequencies, of lesser intrinsic interest, perturb the orbits more and are more readily determined with precision. The precise determination of the M_2 tide therefore requires precise observations of the satellite positions as well as a good coverage of the observations along the satellite orbit. The former requirement is now possible with laser observations of satellites carrying retroreflectors such as the Starlette satellite launched in 1975 by the Centre National d'Etudes Spatiales for tidal and geopotential studies. The second requirement is more readily satisfied with a global network of Doppler stations as the results obtained from Anderle's orbital elements show.

In this paper we discuss results obtained for some ocean tide parameters from two satellites: a Doppler satellite of the Transit navigation network and Geos 1. We compare these results with available ocean models and attempt to combine the two to obtain an improved tide model. We also elaborate and correct certain aspects of our earlier development of the ocean tide potential given in Lambeck *et al.* (1974).

Ocean tide potential

At any point in the ocean the tide component of frequency f_k is expressed by an amplitude $\xi_k^0(\phi, \lambda)$ and phase $\psi_k(\phi, \lambda)$ as

$$\xi_k(\phi, \lambda; t) = \xi_k^0(\phi, \lambda) \cos [2\pi f_k T - \psi_k(\phi, \lambda)],$$

where the phase is with respect to the Greenwich meridian and the time T is expressed in mean solar time. Available ocean tide models refer to the principal frequencies of the Earth–Moon–Sun motion with respect to an ecliptic coordinate frame. On the continents $\xi_k^0 = 0$. To estimate the potential due to this variable layer it is convenient to express ξ_k in spherical harmonics since we need only a very few terms in this expression. Following Lambeck & Cazenave (1973) (but with a slight change in definition so that the phase angles ϵ^+ now vanish for the equilibrium tide) the functions $\xi_k^0 \cos \psi_k$ and $\xi_k^0 \sin \psi_k$ are expanded as follows

$$\xi_k^0 \cos \psi_k = \sum_{s=1}^{\infty} \sum_{t=0}^{s} (a'_{k,st} \cos t\lambda + b'_{k,st} \sin t\lambda)\, P_{st}(\sin \phi),$$

$$\xi_k^0 \sin \psi_k = \sum_{s=1}^{\infty} \sum_{t=0}^{s} (a''_{k,st} \cos t\lambda + b''_{k,st} \sin t\lambda)\, P_{st}(\sin \phi).$$

Then
$$\xi_k = \sum_s \sum_t \sum_{\pm} D^{\pm}_{k,st} \cos(2\pi f_k T \pm t\lambda - \epsilon^{\pm}_{k,s}) P_{st}(\sin\phi) \tag{1}$$

with
$$D^{\pm}_{k,st}\cos\epsilon^{\pm}_{k,st} = \tfrac{1}{2}(a'_{k,st} \mp b''_{k,st})$$
$$D^{\pm}_{k,st}\sin\epsilon^{\pm}_{k,st} = \tfrac{1}{2}(a''_{k,st} \pm b'_{k,st}).$$

Allowing for elastic deformation, the surface load is
$$q_k(\phi,\lambda;t) = \sum_s q_{k,s}(\phi,\lambda;t),$$
$$q_{k,s} = \sum_t \sum_{\pm} \rho_w(1+k'_s)\, D^{\pm}_{k,st}\cos(2\pi f_k T \pm t\lambda - \epsilon^{\pm}_{k,st})\, P_{st}(\sin\phi),$$

where ρ_w is the density of sea water. Outside the Earth the potential becomes
$$\Delta U_k(r) = 4\pi GR \sum_s \left(\frac{R}{r}\right)^{s+1} \frac{1}{2s+1} q_{k,s}$$
$$= 4\pi GR\rho_w \sum_s \sum_t \sum_{\pm} \frac{1+k'_s}{2s+1}\left(\frac{R}{r}\right)^{s+1} D^{\pm}_{k,st}\,.\cos(2\pi f_k T \pm t\lambda - \epsilon^{\pm}_{k,st})\, P_{st}(\sin\phi). \tag{2}$$

The perturbations in the satellite motion due to this potential are found by transforming $(r\phi\lambda)$ into Keplerian equatorial elements of the satellite position at time T, substituting the resulting expression into the Lagrangian planetary equations of motion and integrating these equations with the assumption that the only time-dependent elements are those appearing in the arguments, or the $(\omega\Omega M)$ of the satellite and the frequency f_k of the tidal component investigated (for example, Kaula 1966). The resulting perturbations in i and Ω, the two elements that we analyse for tidal parameters, are (Lambeck *et al.* 1974)

$$\Delta i_{k\;stuv} = A_s D^{+}_{k,st} F_{stu}(i)\, G_{suv}(e)\, \frac{[(s-2u)\cos i - t]}{\dot\gamma^{+}_{k,stuv}} \begin{bmatrix}\sin\\ \cos\end{bmatrix}^{s-t\text{ even}}_{s-t\text{ odd}} \gamma^{+}_{k,stuv},$$

$$\Delta\Omega_{k,stuv} = A_s D^{+}_{k,st} G_{suv}(e) \frac{1}{\dot\gamma^{+}} \left\{\frac{\partial}{\partial i}F_{stu}(i) - (-1)^{s-t}\,\tfrac{3}{2}N\left(\frac{R}{a}\right)^2 \frac{J_2\sin i}{(1-e^2)^2}F_{stu}(i)\right.$$
$$\left.\times\frac{[(s-2u)\cos i - t]}{\dot\gamma^{+}_{k,stuv}}\right\} \begin{bmatrix}\cos\\ \sin\end{bmatrix}^{(s-t)\text{ even}}_{(s-t)\text{ odd}} \gamma^{+}_{k,stuv}$$

with
$$A_s = 4\pi\frac{(1+k'_s)}{2s+1}\frac{GR^2}{a}\left(\frac{R}{a}\right)^s \rho_w \frac{1}{Na^2(1-e^2)^{\frac{1}{2}}\sin i}$$

and
$$\gamma^{+}_{k,stuv} = (s-2u)\,\omega + (s-2u+v)\,M + t(\Omega-\Theta) + 2\pi f_k\, T - \epsilon^{+}_{k,st},$$

where the second term inside the parenthesis in the expression for $\Delta\Omega$ results from the indirect effect of the Earth's oblateness J_2. N is the mean motion of the satellite.

Long-period perturbations, longer than one day, occur only when $\gamma^{\pm}_{k,stuv}$ does not contain the siderial rotation Θ. As the argument $2\pi f_k T$ can be written as $m\Theta + 2\pi f'_k T$, where $m = 2$ for semi-diurnal tides, $m = 1$ for diurnal tides and $m = 0$ for zonal tides, diurnal and semi-diurnal tides will not give rise to long period perturbations for $\gamma^{-}_{k,stuv}$. Only those coefficients $D^{+}_{k,st}$ of the semi-diurnal tide ($m = 2$) and $s, t = 2, 2; 4, 2; 6, 2; \ldots$ will give rise to long-period terms. Other long-period terms will be caused by the coefficients $D^{+}_{k,st}$, with $k = 2$ and $s, t = 3, 2; 5, 2; 7, 2; \ldots$, but now $v = \pm 1$ and the amplitudes of these terms will be smaller than those of previous coefficients by a factor e. Thus unless the satellite orbit is very eccentric these perturbations will be quite small. Only coefficients $D^{+}_{k,st}$, $s, t = 2, 1; 4, 1; 6, 1; \ldots$, of the diurnal tides ($m = 1$) will give rise to long period perturbations with $v = 0$. We note that the amplitudes of the perturbations are proportional to $(R/a)^{s+1}$ so that the coefficients with $s > 4$ will tend to be quite small. Furthermore, we note that the perturbation due to $D^{+}_{k,21}$ or $D^{+}_{k,22}$ has the same dependence on the orbital elements as the perturbation due to the solid tide of the same frequency f_k and that the two cannot be separated. The $D^{+}_{k,42}$ occurs only in the ocean tide potential and due to its different inclination function $F_{stu}(i)$ it can be separated from the leading term in the ocean tide expansion, even though the two have the same frequency, if at least two elements or two different orbits are available. At present the uncertainty in $D^{+}_{k,2t}$ and $\epsilon^{+}_{k,2t}$ probably exceeds that of the uncertainty of the solid tide and we assume that the latter is known. We take $k_2 = 0.30$, the theoretical value for a spherically symmetric Earth computed by Longman (1963), Farrell (1972), Pekeris & Accad (1972) and others. We also assume a zero phase lag; $\epsilon_2 = 0$. The specific dissipation function Q^{-1} of the solid Earth is probably less than $\frac{1}{150}$ at the tidal frequencies so that $\epsilon_2 < 0.5°$. This assumption may introduce an error in the ocean tide phase of about 5° or less.

Table 1. Coefficients of non-normalized ocean tide terms that give long-period orbit perturbations, estimated from numerical models, from satellite solutions and from combined solutions

	tide	D^{+}_{22}/cm	ϵ^{+}_{22}	D^{+}_{42}/cm	ϵ^{+}_{42}
Bogdanov & Magarik	M_2	4.44	54°	0.63	203°
Pekeris & Accad	M_2	4.40	70°	1.4	260°
Zahel (1970)	M_2	4.70	75°	1.35	265°
Hendershott	M_2	5.10	46°	1.20	205°
Bogdanov & Magarik	S_2	1.60	40°	0.20	180°
Zahel (1973)	K_1	6.60	130°	1.7	87°
equilibrium tide	O_1	1.8	270°	0.12	0°
equilibrium tide	K_1	2.5	90°	0.15	0°
equilibrium tide	M_2	4.0	0°	0.25	0°
satellite solution	M_2	3.86	35°	1.26	102°
satellite solution	S_2	2.30	45°	1.8	167°
combined solution (Bogdanov & Magarik)	M_2	4.2	37°	0.9	113°
combined solution (Zahel)	M_2	4.2	43°	0.5	3°

Lambeck *et al.* (1974) analysed several ocean tide models in order to obtain the amplitude coefficients $D^{\pm}_{k,st}$ and phase lags $\epsilon^{\pm}_{k,st}$. The results are given in table 1 for the models of M_2 by Bogdanov & Magarik (1967), Pekeris & Accad (1969), Zahel (1970). Hendershott (1972) gives directly the harmonic coefficients which he estimated from his numerical solution. Other tidal components analysed are S_2 of Bogdanov & Magarik and K_1 of Zahel (1973).

Several tidal frequencies which are small on Earth such as O_1, P_1, K_1 K_2 give rise to significant perturbations in the satellite orbits. No ocean tide models are available for these and in order to obtain estimates of their magnitudes of the orbital perturbations we use an equilibrium tide theory. For a tide raising potential of degree l, order m and frequency f_k

$$\Delta U_{k,lm} = A_k \cos\left[2\pi f_k T + m\lambda\right] P_{lm}(\sin\phi)$$

the equilibrium tide is

$$\xi'_k = \frac{1+k-h}{g} \Delta U_k \, l(\phi, \lambda),$$

where $l(\phi, \lambda)$ is the ocean function. Expanding the latter into spherical harmonics

$$l(\phi, \lambda) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} (a_{ij} \cos j\lambda + b_{ij} \sin j\lambda) P_{ij}(\sin\phi)$$

and substituting it into (2) and rearranging terms gives, writing σ_k for $2\pi f_k T$,

$$\xi'_k = \frac{1}{2}\left(\frac{1+k-h}{g}\right) A_k \sum_{+}^{-} \{(a_{ij}\cos\sigma_k \pm b_{ij}\sin\sigma_k)\cos(m\pm j)\lambda - (a_{ij}\sin\sigma_k \mp b_{ij}\cos\sigma_k)\sin(m\pm j)\lambda\} P_{lm}(\sin\phi)\, P_{ij}(\sin\phi),$$

in which products $\sum_{+}^{-}[\cos(m\pm j)\lambda \text{ or } \sin(m\pm j)\lambda]\, P_{lm}(x)\, P_{ij}(x)$ can be expanded as linear series of polynomials in the form

$$\sum_{u=|m\pm j|}^{l+i} \sum_{+}^{-} Q^{\pm}_{lmiju} P_{u,|m\pm j|}(x)\, \{\cos(m\pm j)\lambda \text{ or } \sin(m\pm j)\lambda\}$$

with
$$Q^{\pm}_{lmiju} = \frac{2u+1}{2} \frac{(u-|m\pm j|)!}{(u+|m\pm j|)!} \int_{-1}^{+1} P_{lm}(x)\, P_{ij}(x)\, P_{u|m\pm j|}(x)\, \mathrm{d}x.$$

The $Q^{\pm}_{lmiju}$ relate to the Clebsch Gordan coefficients used in quantum mechanics. They are non-zero for $l+u+i$ even and for max $(|m\pm j|, |l-i|) < u < l+i$. They have been programmed for us by G. Balmino. The equilibrium ocean tide becomes, after further rearrangement,

$$\xi'_k = \frac{1}{2}\left(\frac{1+k-h}{g}\right) A_k \sum_i \sum_j \sum_u \sum_{+}^{-} \{a_{ij}\cos[\sigma_k + (m\pm j)\lambda] \pm b_{ij}\sin[\sigma_k + (m\pm j)\lambda]\}\, Q^{\pm}_{lmiju} P_{u,|m\pm j|}(\sin\phi). \tag{3}$$

The coefficients in the ocean expansion (2) are given by Balmino, Lambeck & Kaula (1973). In order that mass be conserved, the terms in (3) with $u = 0$ and $|m\pm j| = 0$ should vanish, but from the above expression for diurnal and semi-diurnal tides

$$\xi'_{k,00} = \frac{1}{2}\left(\frac{1+k-h}{g}\right) A_k (a_{lm}\cos\sigma_k - b_{lm}\sin\sigma_k) \frac{(l+m)!}{2l+1}$$

and we have to apply a correction term; or the equilibrium tide becomes

$$\xi_k = \xi'_k - \xi'_{k,00}. \tag{4}$$

To introduce the equilibrium tide into the orbital perturbation theory we compare the equations (3, 4) with (1). That is, we compare those terms in (3) that give $u = s$ and $|m\pm j| = t$ with the corresponding terms in (1). Furthermore, $l = 2$ and as we are interested only in long period perturbations we require that $|m\pm j| = m$; or $j = 0$ for $|m+j| = m$ and $j = 0, 2m$ for $|m-j| = m$. Also $s = 2$ or 4. Then

$$D^{+}_{k,st}\cos\epsilon^{+} = \frac{1}{2}\left(\frac{l+k-h}{g}\right) A_k \sum_i a_{i0}\,(Q^{+}_{2mi0s} + Q^{-}_{2mi0s}),$$

$$D^{+}_{k,st}\sin\epsilon^{+} = 0,$$

$$D^{-}_{k,st}\cos\epsilon^{-} = \frac{1}{2}\left(\frac{1+k-h}{g}\right) A_k \sum_i a_{i(2m)}\, Q^{-}_{2mi(2m)2}$$

$$D^{-}_{k,st}\sin\epsilon^{-} = \frac{-1}{2}\left(\frac{1+k-h}{g}\right) A_k \sum_i b_i\, Q^{-}_{2mi(2m)s}.$$

These expressions are valid for $t = m$, the order of the harmonic perturbing potential.

Analysis of orbital perturbations

Tidal perturbations in the motion of two satellites have been analysed with the objective of estimating ocean tide parameters. The satellites are a Transit satellite (6709201) and Geos 1 (6508901). A third satellite Geos 2 (6800201) which was used in earlier analyses by Douglas *et al.* (1974) and Lambeck *et al.* (1974) has not been used in the present study for two reasons: We consider that the distribution of the elements used by Douglas *et al.* is not sufficiently uniform to separate all the tidal perturbations and that the data used by Lambeck *et al.* is considerably inferior to the data used in the present analysis. For the satellite 6709201, loosely referred to as Transit in the following discussion, we have used the residuals in the inclination and right ascension furnished by R. J. Anderle of the U.S. Naval Weapons Laboratory, Dahlgren. J. G. Marsh of the Goddard Space Flight Center, Greenbelt, provided us with the residuals in the inclination of Geos. 1.

Transit (6709201)

Anderle provided us with residuals in the inclination and right ascension for a period of four years for this satellite. Part of this data set has been published by Buonaguro (1972). Elements have been computed from a 2-day arc and are then extrapolated for 2 days using all known forces acting on the satellite. These extrapolated values are compared with the 'observed' elements deduced from the next 2-day arc. The differences between the observed and extrapolated elements are the residuals used in his analysis. Lambeck *et al.* (1974) used a similar method. Forces acting on the satellite that are modelled in the extrapolation are the Earth's gravity field, the direct lunar and solar attraction, the solid Earth tide with $k_2 = 0.30$ and $\epsilon_2 = 0$, solar radiation pressure and atmospheric drag. Ocean tides have not been included. The orbits are computed in an inertial reference frame. The time used to locate the Earth in an inertial reference frame is an extrapolated function and differs considerably from the true rotation of the Earth. The right ascension of the plane of the orbit must therefore be corrected and this has been done using corrections provided by Anderle. Observational material used in the orbit computations are Doppler observations from an average of about 20 permanent stations well distributed around the world. The accuracy of these observations is better than 10 m in satellite position. The advantage of this system for tidal and other small amplitude perturbation studies is the good orbital coverage obtained for the polar motion (Anderle 1973). A complete discussion of the data, orbit computations and parameter estimation procedure is given by Anderle (1974).

The residuals in the elements reflect changes due to ocean tides and due to other unmodelled or inadequately modelled forces during the two day period over which the extrapolation has been carried out. The total perturbations are therefore given by the cumulative sum of these residuals. For several practical reasons we have been able to analyse only two years of inclination and right ascension residuals. Harmonic analyses of the cumulative effects are given in figure 1 and the ocean components M_2, O_1, S_2 and P_1 clearly rise above the noise level. The theoretical amplitudes and periods are given in table 2, where the former are based on the coefficients given in table 1. Uncertainties in the zonal harmonics will give rise to perturbations with a period equal to the period of ω, or about 125 days. Resonant harmonics of order 13 give rise to perturbations with frequencies of $(\sim 2.3 \text{ days})^{-1}$ compared with a sampling frequency of $(2 \text{ days})^{-1}$, and may give rise to an apparent perturbation of periods 15–20 days due to aliasing. Residual solar radiation pressure perturbations will give systematic errors at the same frequences as the solar tides plus additional terms near 80 days due to the Earth's shadowing effect.

For the M_2 tide whose period is of the order of 10 days compared with the 2-day arcs used by Anderle, a small correction is applied to allow for the averaging of the tidal perturbation during the two day interval. This has been discussed by Lambeck *et al.* (1974).

Geos 1

J. G. Marsh has provided us with inclination residuals for a 620 day interval, data which have also been used by Felsentreger *et al.* (1976) for tidal studies. Data used include camera and Tranet

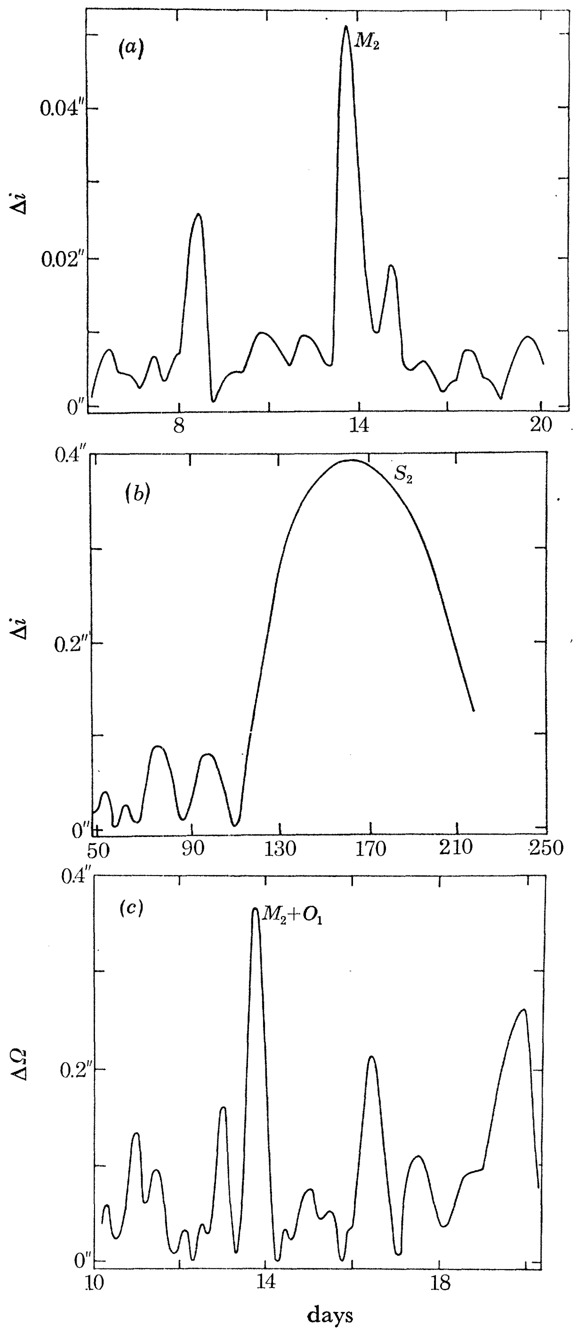

FIGURE 1. (*a*) Harmonic analysis of Δi from Transit for 1971, periods <20 days. (*b*) Harmonic analysis of Δi for Transit for 1971, periods >20 days. (*c*) Harmonic analysis of $\Delta\Omega$ for Transit for 1971, periods >20 days K_1.

Doppler observations and all known forces are modelled in an orbit computation procedure that has been discussed by Douglas *et al.* (1974). Now the inclination residuals correspond to the total tidal perturbation and are comparable to the cumulative residuals discussed above for Anderle's results. The data distribution along the 620-day arc is rather inhomogeneous with consequence that the high-frequency M_2 tide is immersed in noise. From the harmonic analysis of the Δi we

TABLE 2. THEORETICAL AMPLITUDES AND PERIODS OF THE OCEAN TIDE PERTURBATIONS IN THE ORBITS OF THREE SATELLITES

	tide	period/day	amplitude/arcsec Ω	amplitude/arcsec i
Transit	M_2+O_1	14	0.03	
	M_2	14		0.04
	S_2+P_1	170	0.40	
	S_2	170		0.30
Geos 1	M_2	12		0.02
	S_2	56		0.04
	P_1+K_2	80		0.06
	K_1	160		0.20
Starlette	M_2	11		0.02
	M_2+O_1	11		
	S_2	36		

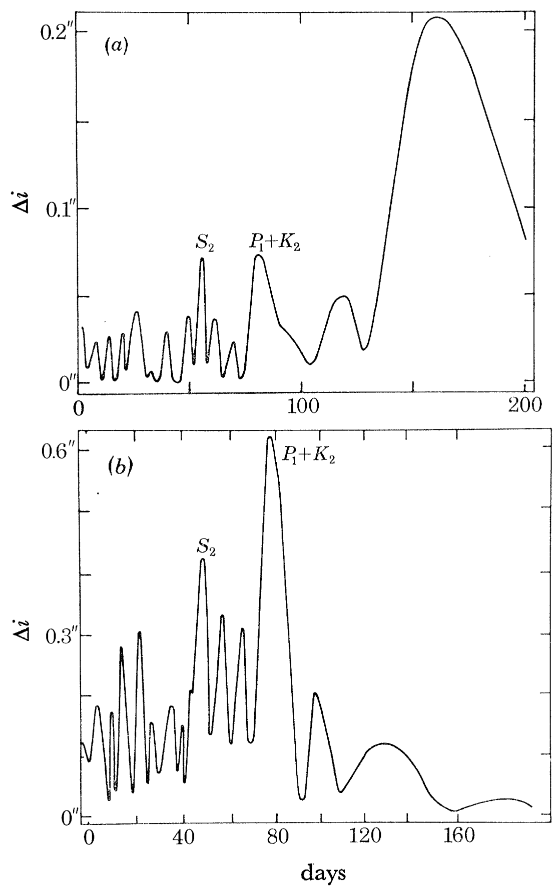

FIGURE 2. (*a*) Harmonic analysis of Δi for Geos 1. (*b*) Harmonic analysis of Δi for Geos 1 after elimination of the tide K_1.

estimate a precision of about $\sigma_{\Delta i} = 0.03''$ and, for want of further information, assume that it is independent of frequency. Figure 2 shows the harmonic analysis of the Δi residuals and table 2 summarizes the theoretical amplitudes and periods. Clearly visible are the S_2, K_1 and P_1 tides as already pointed out by Felsentreger *et al.* (1976) but there is in addition the K_2 tide with nearly the same frequency in the orbital perturbation spectrum as P_1 and of similar amplitude. This component has been overlooked by Felsentreger *et al.* and probably explains their relatively large amplitude obtained for their P_1 ocean tide perturbations.

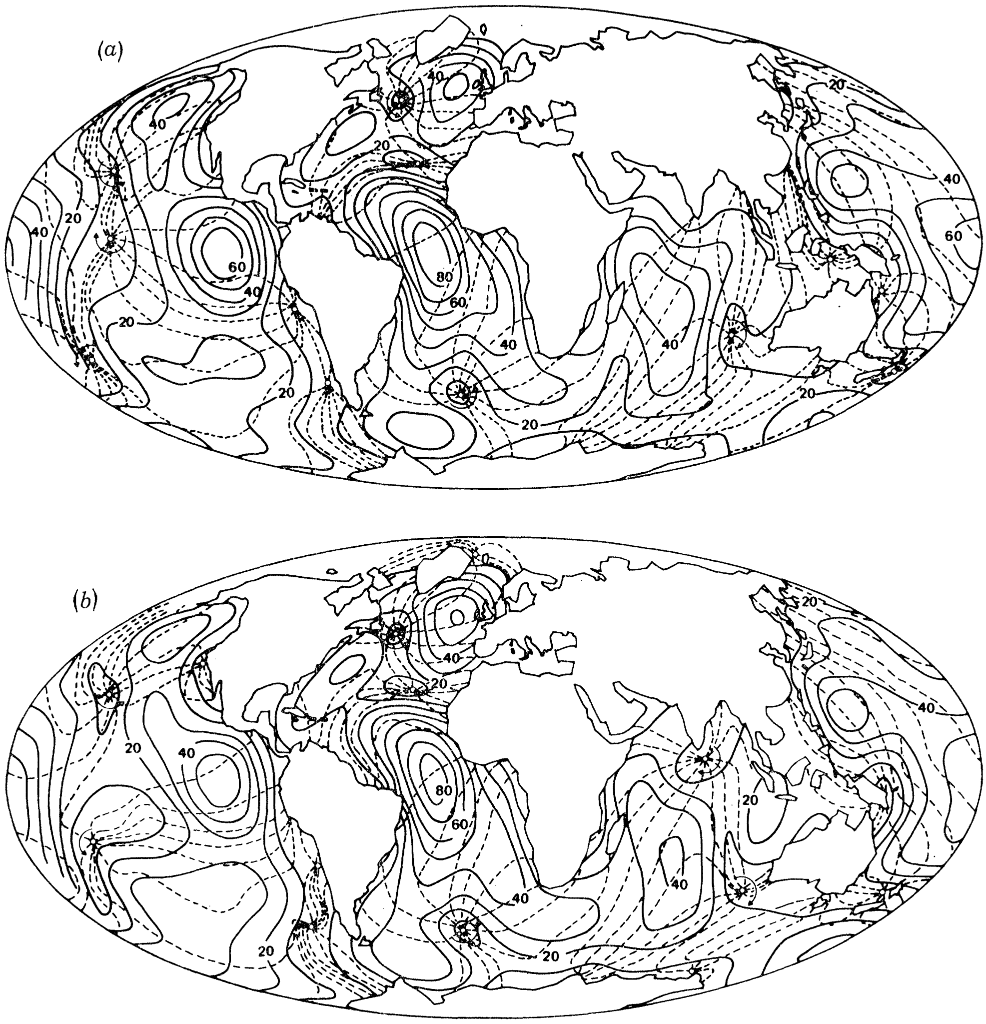

FIGURE 3. (*a*) Numerical model of Bogdanov & Magarike expressed in spherical harmonics up to degree and order 10. (*b*) The combined Bogdanov–Magarike and satellite solution expressed in spherical harmonics up to degree and order 10.

TIDE ANALYSIS

Nominal observation equations, based on average orbital elements, for the ocean tide perturbations are given in table 3. Together with the harmonic analyses shown in figures 1–3 and the theoretical results of table 2 we can summarize the tidal information we hope to extract as follows:

M_2. For the Transit satellite only the inclination can be used directly since the right ascension is also perturbed by the O_1 tide at nearby frequencies and a separation of the two is not possible with

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

the present data. Also, for the polar satellite the nominal observation equations for i and Ω are nearly proportional. Geos 1 elements, due to their uneven time-wise distribution, are only of marginal value for determining M_2. Thus the solution of the degree 2 and four coefficients in the ocean tide will depend essentially on the inclination observations of the Transit satellite.

O_1. Perturbations due to the O_1 tide occur in the right ascension of Transit but they cannot be separated from M_2. Also their amplitudes are generally smaller than those due to the semi-diurnal tide. We could eventually attempt a solution for O_1 by adopting the M_2 solution and then determine O_1 from the right ascension.

TABLE 3. NOMINAL OBSERVATION EQUATIONS OF THE OCEAN TIDE PERTURBATIONS

Transit 670921

$$10^2\Delta i(t) = 1.00''C_{22}^{+} \sin(\sigma t+\epsilon_{22}^{+}) + 0.93''C_{42}^{+} \sin(\sigma t+\epsilon_{42}^{+})$$
$$10^2\Delta\Omega(t) = 0.24''C_{22}^{+} \cos(\sigma t+\epsilon_{22}^{+}) + 0.23''C_{42}^{+} \cos(\sigma t+\epsilon_{42}^{+})$$

Geos 1

$$10^2\Delta i(t) = 0.56''C_{22}^{+} \sin(\sigma t+\epsilon_{22}^{+}) + 0.37''C_{42}^{+} \sin(\sigma t+\epsilon_{42})$$
$$10^2\Delta\Omega(t) = 0.40''C_{22}^{+} \cos(\sigma t+\epsilon_{22}) + 1._{22}''\mathrm{C}_{42}^{+} \cos(\sigma t+\epsilon_{42})$$

Starlette

$$10^2\Delta i(t) = 0.63''C_{22}^{+} \sin(\sigma t+\epsilon_{22}) + 1.14''C_{42}^{+} \sin(\sigma t+\epsilon_{42})$$
$$10^2\Delta\Omega(t) = 0.62''C_{22}^{+} \cos(\sigma t+\epsilon_{22}) + 1.26''C_{42}^{+} \cos(\sigma t+\epsilon_{42})$$

S_2. This solar equivalent to M_2 is clearly evident in the inclination of both Transit and Geos 1 while in the right ascension of Transit it appears together with P_1 (the solar equivalent to O_1) and the two cannot be separated. Furthermore, the observation equations for the S_2 tide in the inclination and right ascension of Transit are also proportional. Thus we solve for S_2 using only the inclinations of the two satellites. The solar atmospheric tide must be corrected for since its contribution is about 15% of that of the ocean tide and this is done by using the tide coefficients given by Lambeck *et al.* (1974). Residual perturbations due to inadequately modelled solar radiation pressure will have the same frequencies as the solar tides so that the results for S_2 may be contamined.

P_1. As for O_1, this tide could be determined simultaneously with S_2 or it could be determined from the inclination of Geos 1 and the right ascension of Transit by using the previously determined parameters for S_2. An additional complication is that for Geos 1 we also have the K_2 tide at a very nearby frequency and a solution for P_1 becomes impossible without further information. Ocean tide models for K_2 do not exist but we assume

$$(D_{22}^{+})_{K_2} = (D_{22}^{+})_{M_2}\,(D_{22}^{+})_{K_2}^{\text{e.t.}}/(D_{22}^{+})_{M_2}^{\text{e.t.}},$$
$$(\epsilon_{22}^{+})_{K_2} = (\epsilon_{22}^{+})_{M_2},$$

and similarly for D_{42}^{+} and ϵ_{42}^{+}. e.t. denotes the values obtained for the equilibrium tide. The Geos 1 residuals can then be corrected for this particular tide component and an approximate solution for P_1 becomes possible. Being of solar origin these results may be somewhat perturbed by residual solar radiation pressure effects.

K_1. Only the Geos 1 satellite is significantly perturbed by this tide and a solution for it is not possible. We can, however, compare the observed amplitude of the perturbations with that expected from the ocean tide model of Zahel (1973) or with the equilibrium tide results.

Due to the slow temporal variation of the orbital elements the numerical coefficients of the observation equations will vary with time and each determination of a residual in i or Ω results in a separate observation equation. Thus a solution for M_2 S_2 and P_1 is possible in the least squares sense for both the second and fourth degree terms in the ocean tide, and the actual solutions show that the separation is possible. We assume equal weight for each observation equation.

The satellite solutions are given in table 1. For both M_2 and S_2 the agreement with the numerical models is reasonable in view of the uncertainties associated with both the satellite values and the ocean tide models. For both tides, the phase of the 4, 2 term is poorly determined. The correlation between the satellite determined coefficients for the M_2 tide remains high, about 0.9, due to the limited contribution of Geos 1 to the solution. For S_2 the correlation coefficients are less than 0.6. The ratio

$$(D_{22}^{+} \cos \epsilon_{22}^{+})_{M_2}/(D_{22}^{+} \cos \epsilon_{22}^{+})_{S_2}$$

estimated from the satellite solution in 1.4, from the Bogdanov & Magarik (1967) models 3.4 while the equilibrium theory gives 2.3. The satellite results are still too uncertain to be able to draw any definite conclusions about their comparison with the numerical model but we have nevertheless attempted a combined solution using the numerical M_2 models of Bogdanov & Magrik and of Zahel (1970). Table 1 summarizes the results for the coefficients 22 and 42 and figure 3 illustrates the ocean tide model of Bogdanov & Magarik before and after the combination, both being expressed in spherical harmonics up to degree and order 10. The combination is carried out in a least squares sense and the correlation matrices are carried along in the solution so that these solutions are valid even if the correlation between the satellite derived coefficients in high.

Conclusions

The ocean tides have been shown to yield significant perturbations in the satellite orbits and some ocean tide parameters have been estimated for M_2 and S_2 that are in reasonable agreement with the results of numerical models of these tides. Improvements can be anticipated when the analysis of the Starlette orbits is completed. Further improvements require additional satellites in different orbits to ensure:

(1) separation of the 22 and 42 terms in the ocean tide expansion,

(2) separation of tides as such M_2 and O_1 that give nearly identical periods in the orbits of the presently available orbits,

(3) eccentric orbits that enable the coefficients 32 and 52 to be estimated in the ocean tide expansion.

We thank R. J. Anderle of the U.S. Naval Weapons Laboratory and J. G. Marsh of the Goddard Space Flight Center for providing the orbital elements.

References (Cazenave *et al.*)

Anderle, R. J. 1971 Refined geodetic results based on Doppler satellite observations, Tech. Report. TR-2889, U.S. Nav. Weapons Lab. Dahlgren. Va.

Anderle, R. J. 1973 Determination of polar motion from satellite observations. *Geophys. Surv.* **1**, 147–161.

Anderle, R. J. 1974 Transformation of terrestrial survey data to Doppler satellite datum. *J. geophys. Res.* **79**, 5333–5348.

Balmino, G., Lambeck, K. & Kaula, W. M. 1973 A spherical harmonic analysis of the Earth's topography. *J. Geophys. Res.* **78**, 478–481.

Bogdanov, K. T. & Magarik, V. A. 1967 Numerical solutions for the world's semi-diurnal (M_2 and S_2) tides. *Dokl. Akad. Nauk. SSSR* **172**, 1315–1317.
Buonaguro, A. F. 1972 Out of plane prediction errors in navigation satellite orbits. *Naval Weapons Lab. Rep.* TR-2880.
Douglas, B. C., Klosko, S. M., Marsh, J. G. & Williamson, R. G. 1974 Tidal perturbations in the orbits of Geos 1 and Geos 2. *Celest. Mech.* **10**, 165–179.
Farrell, W. E. 1972 Deformation of the Earth by surface loads. *Rev. Geophys. Space Phys.* **10**, 761–797.
Felsentreger, T. L., Marsh, J. G. & Agreen, R. W. 1976 Analysis of the solid Earth and ocean tidal perturbations on the orbits of the Geos 1 and Geos 2 satellites. *J. Geophys. Res.* **81**, 2557–2563.
Hendershott, M. C. 1972 The effects of solid Earth deformations on global ocean tides. *Geophys. J. r. astr. Soc.* **29**, 389–402.
Kaula, W. M. 1962 Celestial geodesy. In *Adv. Geophys.* **8**, 191–293.
Kaula, W. M. 1966 *Introduction to satellite geodesy*, p. 124. Waltham, Mass.: Blaisdell.
Kaula, W. M. 1969 Tidal friction with latitude dependent amplitude and phase angle. *Astr. J.* **74**, 1108–1114.
Kozai, Y. 1968 Love's number of the Earth derived from satellites observations. *Publ. Astr. Soc. Jap.* **20**, 24–26.
Lambeck, K. 1973 Determination of Earth and ocean tides from the analysis of satellite orbits. *Symposium on Earth's grav. Field Sec. Var. Posn.*, University of New South Wales, Sydney, pp. 522–528.
Lambeck, K. 1975 Effects of tidal dissipation in the oceans on the Moon's orbit and the Earth's rotation *J. geophys. Res.* **80**, 2917–2925.
Lambeck, K. & Cazenave, A. 1973 Fluid tidal effects on satellite orbit and other temporal variations in the geopotential. *Bull. Groupe Rech. Géod. spatiale* **7**, 20.
Lambeck, K., Cazenave, A. & Balmino, G. 1974 Solid Earth and ocean tides estimated from satellite orbit analyses. *Rev. Geophys. Space Phys.* **12**, 421–434.
Longman, I. M. 1963 A Green's function for determining the deformation of the Earth under surface mass loads. 2. Computations and numerical results. *J. geophys. Res.* **68**, 485–496.
Newton, R. R. 1965 An observation of the satellite perturbation produced by the solar tide. *J. geophys. Res.* **70**, 5983–5989.
Newton, R. R. 1968 A satellite determination of tidal parameters and Earth deceleration. *Geophys. J. r. astr. Soc.* **14**, 505–539.
Pekeris, C. L. & Accad, Y. 1969 Solution of Laplace's equations for M_2 tide in the world oceans. *Phil. Trans. R. Soc. Lond.* A **265**, 413–436.
Pekeris, C. L. & Accad, Y. 1972 Dynamics of the liquid core of the Earth. *Phil. Trans. R. Soc. Lond.* **273**, 237–260.
Smith, D. E., Kolenkiewicz, R. & Dunn, P. J. 1973 Earth tidal amplitude and phase. *Nature, Lond.* **244**, 498–499.
Zahel, W. 1970 Die Reproduktion gezeitenbedingter Bewegungsvorgange im weltozean Mittels des hydrodynamisch-numerischen Verfahrens. *Mitt. Inst. Meereskunde, Univ. Hamburg*, 17.
Zahel, W. 1973 The diurnal K, tide in the world ocean – A numerical investigation. *Pure appl. Geophys.* **109**, 1819–1825.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES